# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – PHYSICS

FIRST SEMESTER – NOVEMBER 2015

### PH 1820 - MATHEMATICAL PHYSICS - I

Date : 11/11/2015

Dept. No.

Max.: 100 Marks

 $4 \times 7.5 = 30$ 

Time : 01:00-04:00

# PART A

# Answer all questions

# 10 x 2 = 20 marks

- 1. Show that the expression  $y = ax^2 + bx$  is reducible to linear form
- 2. Write the algorithm used in modified Euler method
- 3. Sketch the graph defined by  $\overline{z}z = 36$
- 4. Show that  $Ze^{-i\theta}$  acts as an operator when operated on complex number  $Z_2$
- 5. Write the terms contained in  $G = g_{ij} x^i x^j$  for i, j = 3
- 6. Show that Kronecker delta is a mixed tensor of order 2.
- 7. Define norm of a vector
- 8. Show that in  $R^{3}$ , the vectors  $V_1 = (-1,2,1)$  and  $V_2 = (3,1,-2)$  are linearly independent
- 9. Show that all the roots of  $P_n(x) = 0$  are real and lie between -1 to +1.
- 10. Express Hankel functions in terms of Bessel polynomials.

# PART B

# Answer any four questions

- 11. Solve  $\frac{dy}{dx} = y + x^3$  with h= 0.01, y(0) = 1 using Euler's method.
- 12. If  $u = x^3 + 3x^2y 3xy^2 y^3$ , find v such u + iv is analytic.
- 13. Prove that the set all solutions of the differential equation  $a\frac{d^2y}{dy^2} + b\frac{dy}{dx} + cy = 0$  is a vector space.
- 14. i)Show that a skew-symmetric tensor of the second order has  $\frac{n(n-1)}{2}$  different non-zero components

ii) Prove that the sum (or difference) of two tensors of same order and type is again a tensor of the same order and type.

- 15. Evaluate  $\frac{e^z dz}{(z^2 + \pi^2)^2}$ ; |z| = 5
- 16. Show that in the case of Bessel's polynomials  $J_n(x)$  and  $J_{-n}(x)$  are related for integral values of n and these two alone cannot be the solution of it.



#### PART C

#### Answer any four questions

### 4 x 12.5= 50

- 17. Find the roots of the equation  $f(x) = x^3 x 11 = 0$  corrected up to 3 decimal places using Bisection method.
- 18. Using contour integration enaluate  $\int_0^\infty \frac{dx}{dx^6+1}$
- 19. Evaluate the following expressions  $\|\vec{u} + \vec{v}\| \|\vec{u}\| + \|\vec{v}\| \|\vec{u}\| + \|\vec{v}\| \|\vec{u}\| 3\|\vec{v}\|$ 
  - + 3  $\vec{v} \parallel \vec{v}$   $\vec{v} = (1,0,-1,2); \vec{v} = (2,1,3,-1); \vec{v} = (-3,1,0,-5)$
- 20. Derive the components of moment of inertia tensor when a system consisting of number of point masses are in a rotatory motion.
- 21. Using the beta and gamma function evaluate i)  $\int_{0}^{2} \sqrt[3]{8-x^3} x dx$ 
  - ii)  $\int_{0}^{4} x^{\frac{3}{2}} (4-x)^{\frac{5}{2}} dx$  iii)  $\int_{0}^{\frac{\pi}{2}} \frac{1}{\sqrt{tan\theta}} d\theta$
- 22. Evaluate  $\int_{c} (z^2 + 3z) dz$  along a) the circle |z| = 2 from (2,0) to (0,2) in counter clockwise direction b) the straight line (2,0) to (0,2) c) the straight line (2,0) to (2,2) and then from (2,2) to (0,2).

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